COBE Data Spatial–Frequency Analysis and CMB Anisotropy Spectrum.*

Brukhanov A.A., Skulachev D.P., Strukov I.A., Konkina T.V.

February 1, 2008

Space Research Institute, Moscow, Russia
E-mail: istrukov@esoc1.iki.rssi.ru, virgo@relict.iki.rssi.ru

Abstract

We discuss the problem of CMB spectrum corruption during Galactic emission removing. A new technique of spatial–frequency data reduction is proposed. The technique gives us a possibility to avoid a spatial harmonics nonorthogonality. The proposed technique is applied to the two-year COBE DMR sky maps.

We exclude the harmonics with l=7, 9,13, 23 and 25 as having anomalous statistics noise behavior. One shows that procedure do not give systematic errors, if the data are statistically regular.

The spectral parameter of the power spectrum of primordial perturbation $n = 1.84 \pm 0.29$ and quadrupole moment $Q_2 = 15.22 \pm 3.0$ are estimated. The power spectrum estimation results are inconsistent with the Harrison-Zel'dovich $n = 1$ model with the confidence 99%. It is shown a necessity of an increasing a survey sensitivity to reach a more reliable estimation of the cosmological signal.

*accepted to Pis'ma v Astronomicheskij Zhurnal, 1996, v.22, in Russian

1 Introduction.

A large scale cosmic microwave background (CMB) anisotropy carries the information about a primordial metric perturbation amplitude and spectrum, it shape reflects the Universe evolution and structure. To encrypt the information one analyzes a CMB anisotropy spatial spectrum and compares it with a theoretical models with different types of the Universe evolution scenario. A sensitivity rising in the CMB investigations caused a detection of the anisotropy. As a result one may find prerequisites of a more detail examination of the physical conditions involved in the Universe birth and evolve. Being high sensitive and multi-frequent COBE experiment (Smoot et al., 1992, Bennett et al., 1994) gives a possibility to careful expanding of our knowledge about the CMB structure and spectrum.

Unfortunately, the Galaxy radiation predominates even at a millimeter wavelength range. This rises a problem of the radiation filtering. The most Galaxy radiation is coming from rather narrow region located along the Galactic plane. The main method to exclude the radiation is to "cut out" the Galactic plane vicinity and to analyze the rest regions on the sky map (Strukov et al., 1991a, 1991b, Smoot et al., 1992, Bennett et al., 1994, Gorski, 1994, Gorski et al., 1994, Wright et al., 1994b). This method is well known but it works perfectly only if variance data analysis is applied. If we use the Galactic "cut out" in the case of spatial spectrum analysis we lose a spherical functions orthogonality on the rest sphere part. As a result the harmonics begin to influence each to other. The problem may be solved partly using a new basis which is orthogonal over the rest part of the sphere (Gorski, 1994) or is orthogonal only to a monopole and dipole because the excluding of these two components effects the most spectrum modification. The spectra obtained this ways may differ from a real one (Bunn et al., 1994) and may cause the essential different conclusions when interpreting the results.

A modern multi-frequency experiments seemingly allow us to separate the blackbody CMB radiation component from the frequency dependent Galaxy foreground. But variations of the radiation spectral composition prevent the procedure from being performed with a desired accuracy.

We propose the method of the spatial spectrum analysis which allows to reach the most spatial resolution without any loss in the reciprocal harmonics orthogonality. The method implies that the CMB filtering stage is conducted

not before but after the radio map is expanded to spherical harmonics.

There are two filtering stages in the proposed process. The first stage is a spatial filtering one when some components related with the Galactic plane and the Galaxy center are excluded from the total spatial spectrum. By this expedient the Galactic radiation magnitude is essentially reduced. And only after that, during the second filtering stage a "frequency cleaning" procedure is used. At this stage the CMB anisotropy is separated from the Galactic radiation using several frequency channels and some a priori Galaxy radiation model. The both stages nowhere are associated with any signal harmonics orthogonality losses.

The main method difficulty may be found at the second stage and is bound up with the problem of a correct Galactic radiation model selection. But using the first stage filtering we strongly reduce the effect of the model uncertainty on the net result.

As an initial data we used public released COBE DMR 2-years all sky maps from 31.5, 53, and 90 GHz frequency channels. In addition we exploited a 19.2 GHz balloon data kindly provided by the authors of the survey.

2 The Method.

A data reduction process practically always is accomplished by some types of the data censoring like weighing, excluding outliers, corrections for systematic errors, trends and so on. A censor method selection is based on a priori information about the analyzing sample. The censoring is aimed at the improvement of the estimation stability and at the approaching the real data statistic parameters to the used a priori models.

In our case the values on the observed radio map is a sum of a cosmological signal we try to measure, a receiver noise, the Galactic radiation, and perhaps some signal related with neglected systematic effects. It is the cosmological signal that we are interested in, so the other components remain to be carefully estimated, significantly reduced or to be completely excluded. In an ideal case (no Galactic radiation and no systematic effects) the procedure would not cause a significant change in the cosmological signal parameters.

In a Galactic coordinate system one may find a Galactic emission concentrated near the Galactic plane and around the Galaxy center. The radiation may be described using only a few spatial harmonics. If these harmonics

are excluded the Galactic radiation is substantially reduced. In our case we excluded 23 harmonics only. At the same time the rest spatial spectrum consist of 649 harmonics. If a signal is normal distributed the 649 harmonics are quite sufficient for a stable signal spectrum parameter estimation.

To estimate the signal parameter adequately it is absolutely necessary to know (or to estimate) exactly a noise which may be found in any analyzed data. The most problem in the procedure is the presence of anomalous magnitudes in the noise spectrum. If the signal/noise ratio is not good enough the anomalies may cause a significant variation in the parameters to be estimated.

We emphasize that a cause of the anomalies may be some residual systematic errors. In this case it is very difficult to build up a statistical model of the anomaly and the accuracy of the signal estimation may drops below a point that can be tolerated.

If the number of the anomalous harmonics is not large in comparison with the total signal harmonics, the radical solution of the problem is an exclusion of the anomalous harmonics from the analysis. If data are regular, the excluding some anomalous (in the noise spectrum) harmonics is rather safe procedure. That is because the cosmological signal and the noise are Gauss type, and the signal parameters are weakly depended on both the fact of excluding and the criteria of anomalous outliers selection (in our case we rated to anomalous the outliers beyond the 3–sigma).

But if the neglected systematic errors are present in the data, this procedure causes the significant improvement of the sample regularity and causes the increasing of an estimation efficiency. At the same time if the excluding process significantly changes the estimated parameters, it may be a strong reason to assume the data is not regular, and consequently may indicates that it is necessary to use some type of procedure to reduce the irregularity.

It should be noted that using data reduction methods which break a spatial harmonics orthogonality causes a certain spectral smoothing and may reduce or even may camouflage the abnormal amplitudes. In this situation the risk of missing a really existing anomaly is run, and the signal estimation process may be accomplished with some additional errors.

The data reduction procedure we propose is the following. We use COBE A and B channels (Wright et al., 1994a) and produce the sum and the difference radio-maps as a half-sum and a half-difference of the A and B maps respectively. All data on the sum map are a sum of a signal and a noise. All

data on the difference map are only the noise, the signal amplitude is zero.

We exclude from the sum map the monopole and dipole components, and the spectral components cased by the Galaxy emission. Then the sum map is additionally "cleaned" from the Galactic radiation. For this purpose we use 19 GHz and 31 GHz maps to determine the Galactic radiation spectral index. Then the 31 GHz map is subtracted from 53 GHz map, but with a weight corresponding the determined spectral index.

We assume the hypothesis the cosmological signal and the noise are normal distributed. The noise spectrum shape is assumed to be completely determined by the number of measurements in every point on the sky. Based on this we calculate the noise parameters and discover anomalous noise components. Then we use a Monte Carlo simulation to estimate signal magnitude, signal spectrum parameters, and the estimates confidence levels. The latter procedure is applied both to a total analyzed spectrum and to a censored spectrum with anomalous components excluded. In addition we test the determined estimations for stability to the procedure of quadrupole component subtracting.

2.1 Signal Analysis on the Sphere.

The signal on the sphere may be represented as a spherical harmonics expansion $Y_{l,m}(\theta,\varphi)$:

$$\Delta T(\theta,\varphi) = \sum a_l^m Y_{l,m}(\theta,\varphi)$$

In this case the signal variance on the sphere σ^2 is represented as:

$$\sigma^2 = \frac{1}{4\pi}\sum_l \sum_m (a_l^m)^2 = \sum_l \Delta T_l^2$$

where ΔT_l^2 is a l–th spherical harmonic power.

In any real experiment the analyzed signal $a_{l,signal}^m$ is affected by a transfer function W_l (thereafter we use the COBE antenna transfer function (Wright et al., 1994a) as W_l) and some noise $a_{l,noise}^m$ is added:

$$\Delta T(\theta,\varphi) = \sum (W_l a_{l,signal}^m + a_{l,noise}^m) Y_{l,m}(\theta,\varphi)$$

The cosmological signal on the sphere is a sample of a random process. In a standard cosmological scenario the inflation creates adiabatic scalar and

tensor Gauss fields. So the spectrum is assumed to be a sample of a normal random process with independent harmonics, $\langle a_l^m a_k^n \rangle \sim \delta(l,k)\delta(m,n)$.

If initial density perturbation spectrum is $(\delta\rho/\rho)^2 = Ak^n$, one may find for the mean lowest multipoles in expansion $\Delta T(\theta,\varphi)$ (Bond, Efstathiou, 1987):

$$\langle (a_l^m)^2 \rangle \sim \frac{\Gamma(l+(n-1)/2)\Gamma((9-n)/2)}{\Gamma(l+(5-n)/2)\Gamma((3+n)/2)} \qquad (1)$$

A consideration of other physical phenomena (evolution effects, acoustic oscillation, Silk effect etc.) causes a more complicated spectrum (White et al., 1994 and references therein). Hereafter we approximate a real spectrum with the (1). For a quantitative spectrum description we use only two parameters: a power spectrum index n and quadrupole magnitude $Q_{rms}^2 = \langle \Delta T_2^2 \rangle$ as an amplitude.

A correlation function on the sphere $C(\beta) = \langle (\Delta T(\vec{q}_1))(\Delta T(\vec{q}_2)) \rangle$, $\vec{q}_1 \vec{q}_2 = \cos(\beta)$ may be represented in terms of spherical functions:

$$C(\beta) = \frac{1}{4\pi} \sum P_l(\cos\beta) \sum_m (a_l^m)^2$$

Using the spherical harmonic expansion we may calculate the correlation between two maps (for example between 31.5 GHz and 53 GHz maps):

$$\rho(\mathrm{MAP1}, \mathrm{MAP2}) = \frac{\sum_{l,m} a_l^m(1) a_l^m(2)}{\sqrt{\sum_{l,m}(a_l^m(1))^2 \sum_{l,m}(a_l^m(2))^2}}$$

2.2 Spectrum Parameters Analysis.

A cosmological model scenario gives us a prediction of a mean CMB spectrum fluctuation. Experimenters deal with the single sample of the scenario. In addition a noise is always present in a real data. In this connection there is the problem of a signal parameters estimation. The problem was discussed by Bunn et al., 1994 and Sazhin et al., 1995. Here we will not go into the problem, but will touch upon the estimation not only the signal, but the noise parameters too.

The maximum likelihood estimation is known to be the most effective. It may be illustrated if the signal spectrum has a given a priori shape.

Let the spectrum shape is $\langle (a_l^m)^2 \rangle = Q^2 F_{lm}$. If the amplitude is Q, the likelihood function may be described as:

$$f(a_{lm}, Q|F_l) = \prod_{lm} \frac{1}{\sqrt{2\pi Q^2 F_{lm}}} \exp\left(-\frac{a_{lm}^2}{2Q^2 F_{lm}}\right),$$

The maximum likelihood estimation $Q_{\rm ML}$ may be found if solve the equation maximizing the likelihood function:

$$\frac{\partial \ln f(Q|F_l)}{\partial Q^2} = 0$$

$$Q_{\rm ML}^2 = \frac{\sum (a_{lm}^2/F_{lm})}{M}, \qquad M = \sum_{l,m} 1$$

where M is the number of the analyzed harmonics.

The parameter is evident having χ_M^2 distribution with M degree of freedom, $Q_{\rm ML}^2 \sim \chi_M^2$. The amplitude estimation calculated using the variance on the sphere σ^2 is:

$$Q_{\rm POWER}^2 = \frac{\sum \Delta T_l^2}{\sum \langle \Delta T_l^2 \rangle | Q = 1} = \frac{\sigma^2}{\sigma_{th}^2 | Q = 1} \sim \chi_{N_{eff}}^2 .$$

It is clear that: $N_{eff} = \frac{(\sum_{lm} \Delta T_l^2)^2}{\sum_{lm} (\Delta T_l^2)^2} \leq M$. So for the Harrison-Zel'dovich spectrum $N_{eff} \approx 100$, $M \approx 700$ p $l_{max} = 25$. The maximum likelihood estimation accuracy is about $5\,\%$ and the total power estimation is about $14\,\%$.

Using the approach we may determine the signal spatial spectrum parameters in the presence of noise. Maximum likelihood estimation offers to find a minimum of the functional:

$$\sum_{l,m} \ln(W_l^2 \langle (a_l(Q,n))^2 \rangle + \langle (a_{l,noise}^m)^2 \rangle) + \frac{(b_l^m)^2}{W_l^2 \langle (a_l(Q,n))^2 \rangle + \langle (a_{l,noise}^m)^2 \rangle} \quad (2)$$

relative Q and n with given spectrum shape and measured signal and noise sample spectra. Here $\langle (a_l(Q,n))^2 \rangle$ is the model signal spectrum, depending upon desired parameters and described as (1), $(b_l^m)^2$ is the measured spectrum (we describe it further), and $\langle (a_{l,noise}^m)^2 \rangle$ is a model noise spectrum (3),

its amplitude is found using the same approach but from the difference map
.

In addition we are to take into account a procedure of the Galactic radiation "cleaning" and a procedure of a conversion the antenna temperatures into a thermodynamic scale. In all cases the summation is taken over multipoles chosen for the analysis. The necessary conditions for the described procedure is a knowledge of the measured signal and noise spectra.

2.3 COBE Data Noise Analysis.

There are a lot of papers concerning the COBE data noise analysis. Bennett et al., 1994, Lineweaver et al., 1994 discussed the correlation noise properties and showed a weak (0.45%) correlation at 60°. So in practice we may assume the noise is uncorrelated. The noise power estimation may be done using the difference maps.

As a first approximation assume that the noise is white. However the real observations have different accuracy on the sphere. The COBE orbit configuration was so that the points near the ecliptic poles were observed much longer than others. Let us consider the methods to take into account the measurement accuracy is not equal for different points.

Let the noise in every point on the sphere be normal with zero mean and the variance $\sigma^2(\theta,\varphi) = \sigma_0^2/N(\theta,\varphi)$, where $N(\theta,\varphi)$ is a number of measurements in the point with the θ, φ coordinates. Then we may propose the following noise spectrum model on the sphere. If there is a white noise sample $G = \sum n_l^m Y_l^m$, $\langle n_l^m n_{l'}^{m'} \rangle = 0$, $\langle (n_l^m)^2 \rangle = n^2 = const$, the true noise on the sphere is a product of $G(\theta,\varphi)$ (here, and only here n is a magnitude of the noise power spectrum, not a spectrum index), and $1/\sqrt{N(\theta,\varphi)}$:

$$2R(\theta,\varphi) = 1/\sqrt{N(\theta,\varphi)}G(\theta,\varphi) = w(\theta,\varphi)G(\theta,\varphi)$$

According to Peebls (1980), assume $w(\Omega)Y_l^m(\Omega) = \sum_{l'm'} w_{ll'}^{mm'} Y_{l'}^{m'}(\Omega)$, where

$$w_{ll'}^{mm'} = \int d\Omega w(\Omega) Y_{l'}^{m'}(\Omega) Y_l^m(\Omega)$$

For the true noise spherical harmonics expansion $R = \sum a_{l,noise}^m Y_l^m$, coefficients are $a_{l,noise}^m = \sum_{l'm'} w_{ll'}^{mm'} n_{l'}^{m'}$.

Taking into account noncorrelatedness of white noise harmonics a covariation of $a^m_{l,noise}$ may be written as:

$$\langle a^m_{l,noise} a^{m^*}_{l^*,noise} \rangle = \sum_{l'm'} \sum_{l''m''} w^{mm'}_{ll'} w^{m^*m''}_{l^*l''} \langle n^{m'}_{l'} n^{m''}_{l''} \rangle = n^2 \sum_{l'm'} w^{mm'}_{ll'} w^{m^*m'}_{l^*l'}$$

And finally noise power spectrum is:

$$\langle |a^m_{l,noise}|^2 \rangle = n^2 \sum_{l'm'} |w^{mm'}_{ll'}|^2 = n^2 \int d\Omega w^2(\Omega) |Y^m_l(\Omega)|^2 \tag{3}$$

We use not only unit or zero weights, but unrestricted values. Because of that our equation differs from Peebls's one in the squared weights.

Then, using maximum likelihood approach to the difference maps we may calculate the amplitude of the spectrum given above.

The power spectrum model make it possible to analyze the noise spectrum measured from the difference maps. The analysis shows that in general the noise spectrum is well described both the model (3) and the white noise model. It is due to a rather perfect COBE coverage of the celestial sphere. But the same analysis shows for the both models the existence of harmonics with an excess power. So in the noise spectrum of 31 GHz map one may find for $l = 13$ the power is greater than predicted from (3). The power excess is $3.4\sigma_{l=13}$, where predicted multipole power variation σ_l is calculated basing on the model and on the degrees of freedom for the analyzed component. Further we use a censoring of the data to exclude this and similar components from the analysis.

The noise spectral component with $l = 25$ has anomalous high correlation between 31 and 53 GHz maps. After subtracting the Galaxy radiation the excess power is $3.3\sigma_{l=25}$. Components with $l = 25$ were excluded from the analysis too.

We find a significant power excess of several noise components on 53 GHz map. We exclude the components with power excess more than $3\sigma_l$. After that we determine the spectrum parameters for the rest components and the procedure is repeated until the spectrum is free from the abnormal components. Thus we excluded the components with $l = 7$, 23, 9 having the initial power excess of $3.11\sigma_{l=7}$, $2.63\sigma_{l=23}$, and $2.31\sigma_{l=9}$.

In the final analysis we use spectral components from $l_{min} = 2$ to $l_{max} = 22$, except components $l = 7$, 9, 13.

Such censoring effects only a weak power spectrum distortion if the signal amplitude distribution is normal. The matter is discovered by numerical modeling, the results are presented in Chapter 3.

During signal spectrum analysis we ignore a noise spectrum components correlation and use the spectrum shape only. Further modeling shows the same results if we use a pure white noise.

2.4 Selection of the Galactic Radiation Model

Our approach is based on the spatial data filtering which reduces the influence of the Galactic spectrum index uncertainty. The accepted Galactic radiation model for the analyzed frequency region is assumed to involve a synchrotron component with frequency dependence as $T = T_0\nu^{\alpha}$, $\alpha = -3 \pm 0.2$ and a bremsstrahlung component with spectral index $\alpha = -2.1$ (Bennett et al., 1992). If there are the space regions of a different nature of radiation on the line of sight, it may cause a spatial variation of the spectral index α. Let us try to estimate roughly the contribution of such variation if we use some effective but constant value α_0. Let the true value of the spectral index be $\alpha = \alpha_0 + \Delta\alpha$. After the frequency depended part is excluded from the data T_1 T_2 of two frequency channels, one may derive:

$$\delta T = T_1 \left(\frac{\nu_2}{\nu_1}\right)^{\alpha_0+\Delta\alpha} - T_1 \left(\frac{\nu_2}{\nu_1}\right)^{\alpha_0} \approx T_2 \left[\left(\frac{\nu_2}{\nu_1}\right)^{\Delta\alpha} - 1\right] \approx T_2 \Delta\alpha \ln\left(\frac{\nu_2}{\nu_1}\right)$$

The residual signal power caused by the spectral index variation $\sigma^2_{\mathrm{VAR}} = \Delta\alpha^2(\ln\nu_2/\nu_1)^2\sigma^2_{\mathrm{MAP2}}$. For the frequencies 31.5 and 53 GHz the residual RMS is smaller than $0.52\Delta\alpha\sigma_{\mathrm{MAP2}}$. It can be seen that the less is Galactic radiation contribution, the less is the residual RMS on the more high frequency radio map.

It is evident the most Galactic radiation is concentrated near the Galaxy plane and near its center. If the spatial spectrum is represented in Galactic coordinate system the most power may be found in the components with $m = 0$ for the even multipoles (Galaxy plane) and with $m = 1$ for the odd multipoles (Galaxy center). So if we exclude this components the influence of the spectral index variation may be reduced drastically. At the same time this procedure does not break the multipoles orthogonality.

Table 2 shows some results obtained after the spectrum analysis from $l_{min} = 2$ to $l_{max} = 25$.

In addition one may find at the last row the difference between 53 GHz map and the 31 GHz map scaled with a spectral index $\alpha = -2.15$ after spatial filtering. Data are represented in antenna temperatures. One can see the resultant σ_{Sky} may be explained either by a cosmological signal existence or by the spectral index variation within $\Delta\alpha \approx 0.5$.

Unfortunately the available data give no way to estimate the variation with required precision. For more reliable conclusions it is necessary to have the sensitivity many times better than COBE instrument has.

We tried to estimate the spectral index variation crude. For this purpose we use both the COBE data and 19.2 GHz survey (Boughn et al., 1992). We has converted latter data to a COBE beam shape (Wright et al., 1994a). After that we analyzed the frequency dependence of the most intensive spherical harmonics and determined the accuracy of the estimation.

The data are shown in Table 1 where one can see correspondingly: analyzed frequency ranges, spectral index α for desired signal/noise ratio equal to 5, spectral index variation $\Delta\alpha_5$, and the predicted spectral index variance $\Delta\alpha_N$, calculated basing on the noise analysis. In addition there are shown the spectral index α_W and its variation weighted-mean for all harmonics with $m \neq 0$ for $l = 2k$ and with $m \neq 1$ for $l = 2k + 1$.

It may be noticed the absolute value of the spectral index α_{19-31} is regular lower than α_{31-53}. The contribution to the total spectrum of the spectral components with signal/noise ratio equal to 5 is more than 40/found the only spectral component with the spectral index like a synchrotron Galactic radiation has. This is $a_{2,2}$ component with the spectral index $\alpha_{19-31} = -2.86 \pm 0.15$, $\alpha_{31-53} = -2.92 \pm 0.22$.

The different maps correlation analysis gives the mean spectral index estimate in the region $\alpha = -(2.2 \div 2.3)$.

So we may assume the hypothesis the spectral index is constant and its measured variations are caused only by the instrumental noise.

On the basis of the analysis we can conclude for the most significant Galactic spectral components (with the exception of $a_{2,2}$) the Galactic radiation model may be assumed to be one-component, with the radiation cased only by an ionized hydrogen. So the $\Delta\alpha$ variations may be taken as zero.

Now we may determine the measured spectrum

$$b_l^m = \left(k(53) - k(31) \left(\frac{53}{31.5} \right)^{-2.15} \right)^{-1} \left(a_l^m(53\,GHz) - a_l^m(31.5\,GHz) \left(\frac{53}{31.5} \right)^{-2.15} \right) ,$$

where $k(31) = 1./1.025724$, $k(53) = 1./1.074197$ are the scale coefficients to transform the thermodynamic temperatures to antenna temperatures for 31.5 GHz and 53 GHz, $a_l^m(31.5\,GHz)$, $a_l^m(53\,GHz)$ are multipole coefficients for 31.5 GHz and 53 GHz maps.

Unfortunately, the COBE sensitivity is not good enough and the accuracy of the estimation is determined by the instrumental noise. We are pinning our hopes on the "Relict-2" space experiment. "Relict-2" instrument is preparing now, it has an order better sensitivity than COBE has, and it will give the possibility to analyze the Galactic spectral index radiation with high reliability and so to separate the cosmological signal from the Galactic radiation with high accuracy.

3 Results

As a result of the analysis we obtained the spatial spectrum parameters estimates under the assumption that Galactic radiation is one-component. For the quadrupole we found:

$$Q_2 = 15.22 \pm 3.0 \mu\mathrm{K}$$

and for the power spectrum index:

$$n = 1.84 \pm 0.29$$

We analyzed spectral components from $l_{min} = 2$ to $l_{max} = 22$ if $m \neq 0$ for $l = 2k$, and $m \neq 1$ for $l = 2k + 1$, and (due to the noise anomalies) if $l \neq 7, 9, 13$.

The accuracy of the obtained parameters is derived using Monte-Carlo simulation. For all estimates we show RMS errors. The procedures of difference and sum maps producing and of noise parameters determining are included in the Monte-Carlo simulation. The simulation also takes into account the noise, its variations, and a cosmic variance.

We find the total power measured in chosen spatial frequency window as $(70.28\,\mu\mathrm{K})^2$, noise estimate as $(56.76\,\mu\mathrm{K})^2$, and on-sphere signal estimate as $(41.44\,\mu\mathrm{K})^2 \pm (14.68\,\mu\mathrm{K})^2$. Total number of analyzed harmonics is $M = 446$.

Table 3 shows the results. In addition we include into the Table 3 the data obtained with anomalous harmonics, and with excluded quadrupole.

We tested an effect of the harmonics excluding procedure on the signal estimation. In the case of the noise outliers caused by systematic errors and if all harmonics are orthogonal the excluding does not effect the estimation bias, but may only increase the errors in comparison to non-censored data. Simulation shows the Q_2 and n variations are 0.9 and 0.11 respectively.

On the other hand, if the outliers are pure stochastic (i.e. we are dealing with a low-probability noise sample) the noise estimation bias may arise and, as a result may arise the signal estimation bias. We simulated this occasion for the signal and noise with the given spectrum and to examine noise outliers stronger than 2.3σ. After that we calculated the signal estimates both for censored and non-censored spectra.

After 1228 simulations we have found 500 events with the outliers. The corresponding estimates are: $\langle n_{censor} - n_{full}\rangle = 0.07 \pm 0.08$ $\langle Q_{censor} - Q_{full}\rangle = -0.4 \pm 0.83$. It should be noted that the obtained estimates really are upper bounds rather than two-sided limits. It is because we exclude the components, which then are used in some linear combination and thus are additionally normalized. It may, in general, reduce the effect of spectrum censoring.

COBE data analysis shows (see Table 3) that the procedure of the anomalous harmonics excluding effects the significant change in n. If the instrumental noise is completely random the probability to obtain so large difference is smaller than 0.02%. One may see that the applying the anomalous harmonics censoring to COBE data causes a dramatically decreasing of the strange dependence of the results on whether or not the quadrupole is excluded. It may be an indirect support to the necessity of proposed data censoring.

Basing on the obtained parameters we may predict the signal in an experiment like it is conducted in Tenerife (Hancock et al., 1994). Assuming 5.5° beam, 8.1° antennae separation, and 3-point method of observation i.e. $\Delta T = T_0 - 0.5(T_1 + T_2)$, we may calculate the following data for the spectrum (1) type using previously obtained Q_2 and n, excluding components with anomalous noise and applying the spatial filtering: $\sigma_{Tenerif} = 54.82\,\mu\mathrm{K}$.

The same analysis, but without excluding anomalous components shows two estimates:

$\sigma_{Tenerif} = 37.22\,\mu$K with quadrupole included, and: $\sigma_{Tenerif} = 34.47\,\mu$K with quadrupole excluded.

We do not use a 4° binning in the analysis. The binning may decrease the obtained data in some degree. The binning is used in Tenerife experiment and the results are (Hancock S. et al., 1994):

$\sigma_s = 49 \pm 10\,\mu$K for 33 GHz channel, and $\sigma_s = 42 \pm 9\,\mu$K for a sum of the 15 and 31 GHz channels. So the cosmological signal spectrum with the parameters we obtained is in a good agreement with the observations which are more sensitive than COBE for high spatial harmonics.

4 Conclusions.

The systematic effects exclusion problem is met practically in any experiment. In the case of CMB anisotropy observation the Galaxy radiation is the primary effect. The method usually used to suppress the radiation is to "cut off" the Galaxy plane region. It causes the problem of the monopole and dipole exclusion. The latter in its turn effects the additional systematic errors but at the less level.

The orthogonal basis when coupled with the spatial-frequency filtering of the Galactic radiation allows to avoid the systematic errors mentioned above. In addition the proposed approach make it possible to analyze and to exclude more fine effects usually caused errors in spectrum parameters measurements. If an instrumental noise is normal the data censoring could not cause a significant difference between the spectral parameters derived from censored and non-censored data sets. Being detected the difference may be result from either neglected residual effects or a noise nonnormality. Being in the context of the normal distribution function we are forced to exclude the spectral components with an anomalous noise behavior.

The cosmological spectrum parameters we obtained differ in some degree from the parameters derived in previous works used the same initial COBE data. The difference is due to the anomalous noise harmonics exclusion rather than spatial–frequency Galactic radiation filtering. In its turn the harmonics exclusion is possible because of the harmonics orthogonality conservation.

A number of investigators (Table 4) announced the strange end result sensitivity to a quadrupole component. Most likely it is also attributed to anomalous noise harmonics influence.

The results based on 2-year COBE data and obtained by several authors are shown in Table 4. In addition we would like to remind the result of $n = 1.7$, derived by (Hancock S. et al., 1994) after a comparison of 1-year COBE and Tenerife data.

So we obtained the estimate of n more precise and somewhat higher than it is derived by other investigators.

To the best of our knowledge, nobody has investigated the COBE noise (A–B) maps in detail. Gorski et al. (1994) used only the mean noise parameters assumed they are normal. Wright et al. (1994b) used the noise spectrum, but the used basis was not complete orthogonal. Bennett et al. (1994) worked by the help of a correlation function but only on the part of the sphere. In the latter two cases the basis function orthogonality is lost and it is impossible to analyze noise spectrum in detail.

The next result we obtained and that differs from previously published is the rejection of the trivial (with $n = 1$) Harrison-Zel'dovich spectrum on the confidence level of 99%.

As a result there are very unlikely the models with high Λ–term and a more probable are the open models with $\Omega < 1$, if we assume the initial density perturbations are determined by the power law index $n = 1$ (Kamionkowski M., Spergel D.N., 1994). Our result supports the existence of a barionic entropy models. At the same time there are not ruled out the models having $n > 1$ in analyzed scales and having more complicated inflation potential (Starobinsky 1992, White et al., 1994 and references therein).

It must be emphasized that by now the spectrum parameters determining accuracy is not enough to draw more deep inferences. Moreover we can see the obtained results being very sensitive to the accuracy of the noise determining and to the used specific estimation procedure.

We hope the 4-year COBE investigation circle will be accessible in the near future. Unfortunately even the 4-year data will not improve the situation drastically. First, it is necessary to increase the instrumental sensitivity as a minimum an order. This will be reached in the planned experiment "RELICT-2". So the accuracy of the n estimation will be about 5-7% and will be determined the cosmic variance rather than instrumental noise (Sazhin et al., 1995). Second, it is necessary to enhance the anisotropy measurement angle resolution coupled with a sky coverage increasing (Scott et al., 1994). Unfortunately, the modern middle and small scale investigations give us the information only about a few point on the sky and so the results are strongly

contaminated by the sample variance.

Authors thank Boughn S.P., Cheng E.S., Cottingham D.A., and Fixen D.J. for providing the 19 GHz data. The COBE data sets were developed by the NASA/GSFC under guidance of COBE Science Working Group and were provided by the NSSDC. This work was supported partly by RFFI grants N93-02-930, 93-02-931, ISF grants N MO6000, MO6300. We would like to thank A. Klypin and M. Sazhin for helpful discussions and useful comments.

5 References.

Bunn et al. (Bunn E., Hoffman Y., Silk J.)// Astrophys.J. 1994. V.425. P.359.

Bennett et al. (Bennet C.L., Smooth G.F., Hinshaw G., Wright E.L. et.al.)// Astrophys.J.(Letters) 1992. V.396. L7.

Bennett et al. (Bennet C.L., Kogut A., Hinshaw G., Banday A.J. et.al.)// Astrophys.J. 1994. V.436. P.423.

Boughn et al. (Boughn S.P., Cheng E.S., Cottingham D.A., Fixen D.J.)// Astrophys.J. (Letters) 1992. V.391. L49.

Bond, Efstathiou (Bond J.R., Efstathiou G.)// Monthly Not.Roy.Astron.Soc. 1987. V.226, P.655

Gorski (Gorski Krzysztof. M.)// Astrophys.J.(Letters) 1994. V.430. L85.

Gorski et al. (Gorski K.M., Hinshaw G., Banday A.J., Bennet C.L. et.al.)// Astrophys.J.(Letters) 1994. V.430. L89.

Kamionkowski, Spergel (Kamionkowski M., Spergel D.N.)// Astroph.J., 1994, V.432, P.7-16

Lineweaver (Lineweaver C.H., Smooth G.F., Bennet C.L., Wright E.L. et.al.)// Astrophys.J. 1994. V.436. P.452.

Peebls (Peebls P.J.E.)// The Large-Scale Structure of the Universe, Princeton: Princeton Univ.Press, 1980.

Wright et al. (Smooth G.F., Kogut A., Hinshaw G., Tenorio L. et.al.)// Astrophys.J. 1994. V.420. P.1

Wright et al. (Wright E.L., Smooth G.F., Bennet C.L., Lubin P.M.)// Astrophys.J. 1994. V.436. P.443.

White et al. (White M., Scott D., Silk J.)// Ann. Rev. Astron. and Astrophys. 1994. V.32. P.319.

Hancock et al. (Hancock S., Davies R.D., Lasenby A.N., Gutierrez de la Cruz C.M. et.al.)// Nature. V.367. P.333.

Sazhin et al. (Sazhin M.V., Brukhanov A.A, Strukov I.A., Skulachev D.P.) // Pis'ma v astronomichesky zhurnal, 1995, v.21, p.403 (in Russian, in English see Astronomy Letters (Soviet Astronomy Letters), 1995, v.21, p.358-365)

Scott et al. (Scott D., Srednicki M., White M.)// Astrophys.J.(Letters) 1994. V.421. L5.

Smoot et al. (Smoot G.F., Bennett C.L., Kogut A., Wright E.L., et.al.)// Astrophys.J.(Letters) 1992. V.396. L1.

Starobinsky (Starobinsky A.A) // Pis'ma v zhurnal experim. i teoret. fiziki, 1992. v.55. p.477 (in Russian)

Strukov et al. (Strukov I.A., Brukhanov A.A, Skulachev D.P., Sazhin M.V.)// Pis'ma v astronomichesky zhurnal 1992a. v.18. p.387 (in Russian, in English see Soviet Astronomy Letters, 1992, v. 18, p. 153)

Strukov et al. (Strukov I.A., Brukhanov A.A, Skulachev D.P., Sazhin M.V.)// Monthly Not. Roy. Astron. Soc. 1992b. V.258, P.37p.

Sugiyama, Silk (Sugiyama N., Silk J.)// Phys.Rev.Letters. 1994. V.73. P.509.

6 Table Caption

Table 1. Mean spectral indexes and its variations determined for spherical harmonics at different frequency ranges. The first column – the frequency ranges where the spectral index α is analyzed. The second column – spectral index determined from the most significant spectral components with the signal/noise ratio lager than 5, α_5. The third column – its variation, $\Delta\alpha_5$. The forth column – spectral index variance predicted from noise analysis, $\Delta\alpha_N$. The fifth column – the mean weighted spectral index α_W for all harmonics with $m \neq 0$ for $l = 2k$, and $m \neq 1$ for $l = 2k+1$. The sixth column – its variance $\Delta\alpha_W$.

Table 2. Comparative result of an influence of spatial filtering (i.e. excluding the components with $m = 0$ for $l = 2k$ and $m = 1$ for $l = 2k$ from $l = 2$ to $l = 25$ inclusively) to a signal amplitude for different frequencies. The last row shows the signal amplitude after the spatial – frequency filtering assuming the Galactic radiation spectral index as corresponded to ionized hydrogen. $\sigma_{(A+B)/2}$ – amplitude, calculated as a half-sum of two maps (i.e. signal plus noise), $\sigma_{(A-B)/2}$ – noise, calculated as a half-difference of two maps, σ_{Sky} – signal amplitude estimation on the sky.

Table 3. Spectrum analysis after spatial–frequency Galactic radiation filtering. The influence of quadrupole excluding is shown. In addition it is shown the results after noise anomalous harmonics are excluded. The first row is corresponded to spectrum from $l = 2$ to $l = 22$ with the abnormal harmonics excluded. The second row – the same but in addition the quadrupole is excluded. The third row shows the results if all harmonics from $l = 2$ to $l = 25$ are used. The forth row shows in this case the influence of quadrupole exclusion. σ^2_{A+B} – total measured power within given spatial window, σ^2_{A-B} – noise power estimation obtained within the same window from a difference map, σ^2_{Sky} – cosmological signal power estimation, ($\sigma^2_{Sky} = \sigma^2_{A+B} - \sigma^2_{A-B}$), Q_2, n – estimations of parameters for spectrum (1), M – total number of analyzed harmonics. In all cases we assume that $m \neq 0$ for $l = 2k$ and $m \neq 1$ for $l = 2k + 1$.

Table 4. Results of CMB anisotropy spatial spectrum analysis ob-

tained by different authors basing on COBE 2-years data. For the comparison our result is shown. It is shown how a quadrupole excluding does influence on the end result.

Table 1:

frequency (GHz)	α_5	$\Delta\alpha_5$	$\Delta\alpha_N$	α_W	$\Delta\alpha_W$
19.2 – 31.5	-2.13	0.36	0.38	-2.12	0.69
31.5 – 53.0	-2.27	0.28	0.36	-2.19	0.76

Table 2:

frequency (GHz)	spatial filtering	$\sigma_{(A+B)/2}$ (μK)	$\sigma_{(A-B)/2}$ (μK)	σ_{Sky} (μK)
31.5	yes	641.	95.92	634.70
	no	254.79	94.52	239.83
53.	no	191.64	33.04	188.80
	yes	83.32	32.20	76.86
frequency "cleaning" $\alpha = -2.15$	yes	50.78	45.44	22.67

Table 3:

NN	Quadrupole analysis	σ^2_{A+B} μK^2	σ^2_{A-B} μK^2	σ^2_{Sky} μK^2	Q_2 μK	n	M
1	yes	$(70.28)^2$	$(56.76)^2$	$(41.44)^2 \pm (14.68)^2$	15.22 ± 2.9	1.84 ± 0.29	446
2	no	$(68.92)^2$	$(56.47)^2$	$(39.51)^2 \pm (14.65)^2$	15.55 ± 3.8	1.81 ± 0.37	442
3	yes	$(82.91)^2$	$(74.19)^2$	$(37.03)^2 \pm (17.48)^2$	18.03	1.31	649
4	no				20.3	1.12	645

Table 4:

Author	n	Q_{rms-PS}
Quadrupole included		
Bennett et al.(1994)	$1.42^{+0.49}_{-0.55}$	$12.8^{+5.2}_{-3.3}$
Gorski et al.(1994)	$1.22^{+0.43}_{-0.52}$	$17.0^{+7.5}_{-5.2}$
Wright et al.(1994b)	$1.39^{+0.34}_{-0.39}$	
this work	1.84 ± 0.29	15.22 ± 2.9
Quadrupole excluded		
Bennett et al.(1994)	$1.11^{+0.60}_{-0.55}$	$15.8^{+7.5}_{-5.2}$
Gorski et al.(1994)	$1.02^{+0.53}_{-0.59}$	$20.0^{+10.5}_{-6.5}$
Wright et al.(1994a), $l = 3 - 30$	$1.25^{+0.40}_{-0.45}$	
Wright et al.(1994b), $l = 3 - 19$	$1.46^{+0.39}_{-0.44}$	
this work	1.81 ± 0.37	15.55 ± 3.8